SUBMISSION TO CAUSALITY CHALLENGE 2011

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ABSTRACT. We report the methods used to obtain our solution for the simulation part of the causality Challenge 2011.

1. The transfer entropy metric

Transfer entropy is a model-free measure of predictive information flow [1] suited to investigate directed statistical dependencies (influences) between two time series without explicit knowledge about the inetraction mechanism. Schreiber [2] introduced transfer entropy from a process X to a process Y as a deviation from the following generalized Markov property (given that the two processes can be approximated by markov chains):

$$p(y_{t+1}|\mathbf{y_t^n}, \mathbf{x_t^m}) = p(y_{t+1}|\mathbf{y_t^n}).$$

For our purposes we measure transfer entropy from X to Y by:

$$TE\left(X \to Y\right) = S\left(\mathbf{y}_{t}^{d_{y}}, \mathbf{x}_{t}^{d_{x}}\right) - S\left(y_{t+u}, \mathbf{y}_{t}^{d_{y}}, \mathbf{x}_{t}^{d_{x}}\right) + S\left(y_{t+u}, \mathbf{y}_{t}^{d_{y}}\right) - S\left(\mathbf{y}_{t}^{d_{y}}\right) \; .$$

where t is a discrete valued time-index and u denotes the prediction time, a discrete valued time-interval. $\mathbf{y}_t^{d_y}$ and $\mathbf{x}_t^{d_x}$ are d_x - and d_y -dimensional delay vectors constructed by Takens delay embedding [3]. Entropy estimations were performed using the Kraskov-Stoegbauer-Grassberger estimator [4], implemented in the TREN-TOOL toolbox (www.trentool.de).

2. Analysis procedure

Raw data were initially cut into 14 epochs of 400 samples. Subsequently, we added to each trial the next 399 samples in the time series to create longer episodes for transfer entropy analysis. The delay parameter u for specifying the potential interaction delay between the time series was scanned from 2 to 12 samples in steps of one sample, and the value with the maximum difference of the test statistic for oroginal and surrogate data was chosen. For each choice of u, embedding parameters were optimized by the Ragwitz criterion for stochastically driven time series [5]. Significance of an interaction was assessed by means of a test against surrogate data as described in [6, 7]. We set a value of '1' in the results vector if either only the interaction from signal 1 to signal 2 was significant, or if the p-value for signal 1 was lower than for signal 2, in case both interactions were significant. For the

1

inverted cases we set a '-1'. If none of the interactions of a pair reached significance we set the result to '0'.

Results were stored in a variable named 'Results' as a 1x1000 vector, in a file named 'ChallengeAnalysis_V_ResultsVector.mat'.

References

- J. Lizier and M. Prokopenko. Differentiating information transfer and causal effect. Eur. Phys. J. B, 73:605-615, 2010.
- [2] Schreiber. Measuring information transfer. Phys Rev Lett, 85(2):461–464, Jul 2000.
- [3] Floris Takens. Dynamical Systems and Turbulence, Warwick 1980, volume 898 of Lecture Notes in Mathematics, chapter Detecting Strange Attractors in Turbulence, pages 366–381. Springer, 1981.
- [4] Alexander Kraskov, Harald Stgbauer, and Peter Grassberger. Estimating mutual information. Phys Rev E Stat Nonlin Soft Matter Phys, 69(6 Pt 2):066138, Jun 2004.
- [5] M. Ragwitz and H. Kantz. Markov models from data by simple nonlinear time series predictors in delay embedding spaces. *Physical Review E*, 65:056201, 2002.
- [6] R. Vicente, M. Wibral, M. Lindner, and G. Pipa. Transfer entropy-a model-free measure of effective connectivity for the neurosciences rid e-1566-2011. *Journal of Computational Neuroscience*, 30(1):45–67, February 2011.
- [7] M. Wibral, B. Rahm, M. Rieder, M. Lindner, R. Vicente, and J. Kaiser. Transfer entropy in magnetoencephalographic data: Quantifying information flow in cortical and cerebellar networks rid e-1566-2011. Progress In Biophysics & Molecular Biology, 105(1-2):80-97, March 2011.

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